Inferring the individual nature of Bayesian learning under multiple forms of uncertainty

Christoph Mathys1,2,† · Jean Daunizeau1,4 · Kay H. Brodersen1,3 · Sandra Iglesias1 · Karl J. Friston4 · Klaas E. Stephan1,4
1 Laboratory for Social and Neural Systems Research (SNS Lab), University of Zurich 2 Institute for Biomedical Engineering, ETH Zurich 3 Department of Computer Science, ETH Zurich 4 Wellcome Trust Centre for Neuroimaging, University College London

1 Introduction
Probability theory formally prescribes an optimal way for learning about the environment from sensory information: sequential updating of beliefs according to Bayes’ theorem. This principle can be extended to hierarchical Bayesian models when dealing with higher-order uncertainty induced by the time-varying structure of the environment. While optimal from the perspective of probability theory, Bayesian belief updating requires evaluating complicated integrals which are not tractable analytically and difficult to evaluate in real time. Recently, however, theoretical advances have enabled computationally efficient approximations to exact Bayesian inference during learning. Here, we focus on a recent derivation of reinforcement learning from Bayesian principles [1] which rests on a generative hierarchical model of the environment and its (in)stability. This model (i) provides analytic and computationally highly efficient update equations (ii) allows for on-line estimates of the model’s states and parameters and (iii) includes a mechanism for time-varying encoding of the precision of beliefs which may correspond to the modulatory effects of dopamine as proposed by [2]. Here, we combine this perceptual model with a model for subjects’ responses, such that decisions are informed by current beliefs about the state of the world.

2 The generative model: a hierarchy of Gaussian random walks
An agent is taken to receive a sequence of inputs \( u^{(i)} \), \( u^{(j)} \). It uses these to make inferences on a hierarchy of hidden states \( x_{2}, \ldots, x_{n} \) of its environment. While \( x_{2} \) is binary, all higher states are continuous. Continuous states change by performing Gaussian random walks that are hierarchically coupled with one state's step size determined by the next highest state.

3 A novel variational inversion leads to closed-form update equations based on learning rate and prediction error
We invert the generative model variationally using a mean field approximation and a novel quadratic approximation to the variational energy that defines the posteriors of the continuous states. This permits us to derive closed-form Markovian update equations for the posterior expectations of all states. These update equations are structurally similar to those of reinforcement learning.

4 The response model
The response model is determined by the parameters \( \omega, \theta \) and \( \phi \) and the belief state \( x_{i} \).

5 Multiple forms of uncertainty: perceptual and decision noise

6 Parameter estimation and application to behavioral data

7 Conclusions
We have provided a novel Bayesian model for individual learning under uncertainty, showing that parameter estimates can be recovered under simultaneous perceptual and decisional uncertainty. The application to behavioral data has demonstrated the potential of our approach for detecting subtle individual differences in learning that would escape conventional analyses.